

B.Sc. Semester - 1 (CBCS) Examination
Oct/Nov. -2019 (OLD COURSE)
MATHEMATICS (CORE)

Time: 2:30 Hours

Marks: 70

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer any one of the following questions.**[07]**

- (1) State and prove Roll's mean value theorem and explain it geometrically.
- (2) For $x > 0$ show that $\frac{x}{1+x^2} < \tan^{-1}(x) < x$.

Q.1 (B) Answer any one of the following questions.**[04]**

- (1) Separate the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x - 5$ is increasing or decreasing.
- (2) Find Maclaurin's series expansion of the function $f(x) = e^x$.

Q.1 (C) Answer any Three of the following questions.**[03]**

- (1) State Cauchy's Mean value theorem.
- (2) $LUB \{ -n : n \in \mathbb{N} \} = \underline{\hspace{2cm}}$.
- (3) Write Maclaurin's series expansion of $\cos x$.
- (4) Write coefficient of x^{10} in Maclaurin's series expansion of $\sin x$.
- (5) If $f(x) = \sin x$, $x \in [0, \pi]$ then find value of c in Roll's theorem.

Q. 2 (A) Answer any one of the following questions.**[07]**

- (1) Evaluate : $\lim_{x \rightarrow \frac{\pi}{2}} \left(\cot^2 x - \frac{1}{x^2} \right)$.
- (2) Define linear differential equation and solve $y \log y \, dx + (x - \log y) \, dy = 0$.

Q.2 (B) Answer any one of the following questions.**[04]**

- (1) Evaluate : $\lim_{x \rightarrow 1} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.
- (2) Solve : $(xy^2 + x) \, dx + (yx^2 + y) \, dy = 0$.

Q.2 (C) Answer any Three of the following questions.**[03]**

- (1) $\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x} = \underline{\hspace{2cm}}$.
- (2) Evaluate : $\lim_{x \rightarrow 0} (x^2 \log ax)$
- (3) Find I. F. of the differential equation $(x + 1) \frac{dy}{dx} - y = e^{3x}$.
- (4) Write degree of the differential equation $\frac{d^3 y}{dx^3} = \left[x + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{5}{2}}$.
- (5) Evaluate : $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right)$.

Q.3 (A) Answer any one of the following questions.**[07]**

- (1) State and prove necessary and sufficient condition for a differential equation to be exact.
- (2) Solve : $(\sin x \sin y + \sec^2 x) \, dx + (\tan^2 y - \cos x \cos y) \, dy = 0$.

Q.3 (B) Answer any one of the following questions.**[04]**

- (1) Solve : $y = 2px + y^2 p^3$.
- (2) Solve : $p(p + y) = x(x + y)$.

Q.3 (C) Answer any Three of the following questions.**[03]**

- (1) Solve the differential equation : $y = p x + p$.
- (2) Define Exact differential equation.
- (3) Write general form of Lagrange's differential equation.
- (4) Determine whether the differential equation $(x - 2y) \, dx + (y - 2x) \, dy = 0$ is exact or not.
- (5) Write an example of a Bernoulli's differential equation which is not linear.

Q.4 (A) Answer any one of the following questions.**[07]**

- (1) In usual notation show that if $f(D) = (D - a)^r g(D)$, then $\frac{1}{f(D)} e^{ax} = \frac{x^r e^{ax}}{r! g(a)}$, Where $g(a) \neq 0$.
- (2) Solve : $\frac{d^2 y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$.

Q.4 (B) Answer any one of the following questions.**[04]**

- (1) Write general form of linear differential equation of n^{th} order, define its auxiliary equation and complementary function.
- (2) Solve : $\frac{d^3 y}{dx^3} - 3 \frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} - y = 0$.

Q.4 (C) Answer any Three of the following questions.**[03]**

- (1) Evaluate : $\frac{1}{D} \sin 2x$.
- (2) Write expansion of $(1 - D)^{-1}$ in terms of D .
- (3) $\frac{1}{D^2 - 1} \cos 5x =$ _____ .
- (4) $\frac{1}{D^2 + 1} \cos x =$ _____ .
- (5) Find complementary function of the equation $(D^2 + 2D + 1)y = 2x + x^2$.

Q.5 (A) Answer any one of the following questions.**[07]**

- (1) Solve : $(x^2 D^2 + 4xD + 2)y = e^x$.
- (2) Solve : $(x^2 D^2 + 1)y = 3x^2$.

Q.5 (B) Answer any one of the following questions.**[04]**

- (1) Solve : $(x^2 D^2 + 3xD + 1)y = 0$.
- (2) In usual notations prove that $\frac{1}{f(\theta)} x^m = \frac{1}{f(m)} x^m$.

Q.5 (C) Answer any Three of the following questions.**[03]**

- (1) Write general form of homogeneous linear differential equation with variable coefficients.
- (2) If $x \frac{d}{dx} = \theta$, then $x^5 \frac{d^5}{dx^5} =$ _____.
- (3) Solve : $(x^2 D^2)y = 0$.
- (4) $\frac{1}{D - 1} e^x =$ _____.
- (5) $\frac{1}{\theta^2 + 6\theta + 5} x^5 =$ _____.
